On Efficient Rate Design

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Efficient Rate Design

A rate design is a relationship of the rates for a single category of mail to an hedonic property of the mail.

Example: Single-Piece Bound Printed Matter. The rate is a function of the weight per piece.

In general, the U.S. Postal Service (USPS) tariff is an hedonic price function that relates many categories of mail to a number of hedonic properties such as weight per piece, distance transported, speed of delivery, pro-sortation level, etc.

At present USPS (and, previously, the PRC) depend upon rules of thumb to design rates. A typical rule scales the marginal cost for an individual category to derive rates that recover a specified contribution to offset USPS' total non-variable costs (aka "institutional" cost).

An efficient rate design would recover the same contribution with the least loss of welfare. This is an extension of the principle that underlies Ramsey-Boiteux pricing.

The paper presents a derivation of two simple propositions that govern the design of efficient rates.

Application of the propositions is illustrated with an example.

The propositions and example demonstrate that USPS' rules of thumb are likely to leave "free lunches" in the form of many small welfare gains that can be recovered with no loss in USPS' net revenue.

The Demand Model

The demand model is a natural generalization of a linear demand function, $Q = \alpha + \beta P$, to accommodate volumes and prices that are continuous functions of an hedonic property over a specified range.

The demand function is represented as a linear integral equation:

$$Q(u) = \alpha(u) - \beta \int_0^1 [I(u,v) - (\gamma/\beta)K(u,v)]P(v)dv \text{ for } u \in [0,1].$$

u, v - indices of an hedonic property with range [0,1].

$Q(u)$ - the demand volume function.

$P(v)$ - the hedonic price function.

$I(u,v)$ - the identity function, i.e., $\int_0^1 I(u,v)P(v)dv = P(u)$.

$K(u,v)$ - a two-variable function describing the relative cross effect of $P(v)$ on $Q(u)$. The strength of a cross effect would typically increase as $u \rightarrow v$, i.e., as two products have closer hedonic properties as indexed by u and v.

$\alpha(u)$ - the demand volume function intercept, i.e., $Q(u)$ for $P(u) = 0 \; \forall \; u \in [0,1]$.

$-\beta \int_0^1 I(u,v)P(v)dv = -\beta P(u)$ - the own-price effect, the effect on demand of changes in the own-price alone. We would expect the effect to be negative, so $\beta > 0$.

$\int_0^1 \gamma K(u,v)P(v)dv$ - the combined cross-price effect, the effect on demand of changes in the prices of all levels of the hedonic property. We would expect cross-price effects to predominantly be substitution effects, so $\gamma > 0$ if $K(u,v) \geq 0$.

The Cost Model

The cost model is a natural generalization of a linear cost function $C = C_f + MQ$ to accommodate volumes and marginal costs that are continuous functions of an hedonic property over a specified range.

The cost model is also represented as a linear integral equation:

$$C = C_f + \int_0^1 M(u)Q(u)du.$$

C - the total cost attributed to the entire category of mail.

C_f - the fixed component of C, usually including a stipulated portion of the post's total institutional costs, i.e., costs that are non-variable at the margin.

$M(u)$ – the marginal cost function.

$\int_0^1 M(u)Q(u)du$ - the "volume-variable" cost of the category of mail, i.e., the component of cost that varies with the quantity of mail when the cost function is approximated by a linear equation.

The Algebra

The paper presents an application of algebraic methods for solving linear integral equations. The methods are described in a self-contained mathematical Appendix.

The methods apply to one- and two-variable functions represented as linear and quadratic forms of an assumed column vector function $f(u)$ defined over the unit interval, $u \in [0,1]$. The elements of $f(u)$ are elementary real-valued functions. They must be linearly independent and their cross-products must be integrable over the unit interval. For example, $f(u)' = [1\ u\ u^2 \ldots\ u^n]$.

One-variable functions: Linear forms. Example, $Q(u) = f(u)'q$, with q a real vector.

Two-variable functions: Quadratic forms. Example, $K(u,v) = f(u)'Kf(v)$, with K a real square matrix.

The essential reproductive property of the algebra is that multiplication and division leave one- and two-variable functions that remain linear and quadratic forms of $f(u)$.

The algebra has many similarities to matrix algebra and allows us to solve a linear integral equation in a way that is similar to the way that we would solve a matrix equation. For example, the inverse demand function is obtained by solving the linear integral demand equation for $P(u)$:

$$P(u) = (1/\beta)\int_0^1 [I(u,v) + B(u,v)]\,[\alpha(v) - Q(v)]dv \text{ for } u \in [0,1].$$

$I(u,v)$ and $B(u,v)$ are two-variable functions with matrices that can be calculated as described in the Appendix.

An Outline of the Math

At the heart of the paper is an application of the algebra to manipulate the solution to a "classical" problem from the calculus of variations.

The problem is to find the function $P(u)$ that maximizes social welfare subject to the condition that revenue covers cost (including C_f). Social welfare is defined as the sum of consumers' and producer's surpluses.

The math consists of formulating an integral equation for welfare, forming a Lagrangian from the equation and the revenue condition, and then, deriving the Euler equation for the Lagrangian. The Euler equation is a necessary condition for the maximization of an integral equation such as the Lagrangian.

The Euler equation is manipulated algebraically to yield two remarkably simple propositions describing the essential properties of an efficient price function, $P(u)$, and the corresponding efficient quantity (volume) function, $Q(u)$.

Proposition 1: The efficient price function is a weighted average of the marginal cost function and the zero-volume price function.

Proposition 2: The efficient volume function is proportional to the volume function corresponding to marginal cost pricing.

Finally, it is shown how the Propositions may be employed to derive the efficient price function, $P(u)$, and volume function, $Q(u)$, for a predetermined cost contribution C_f.

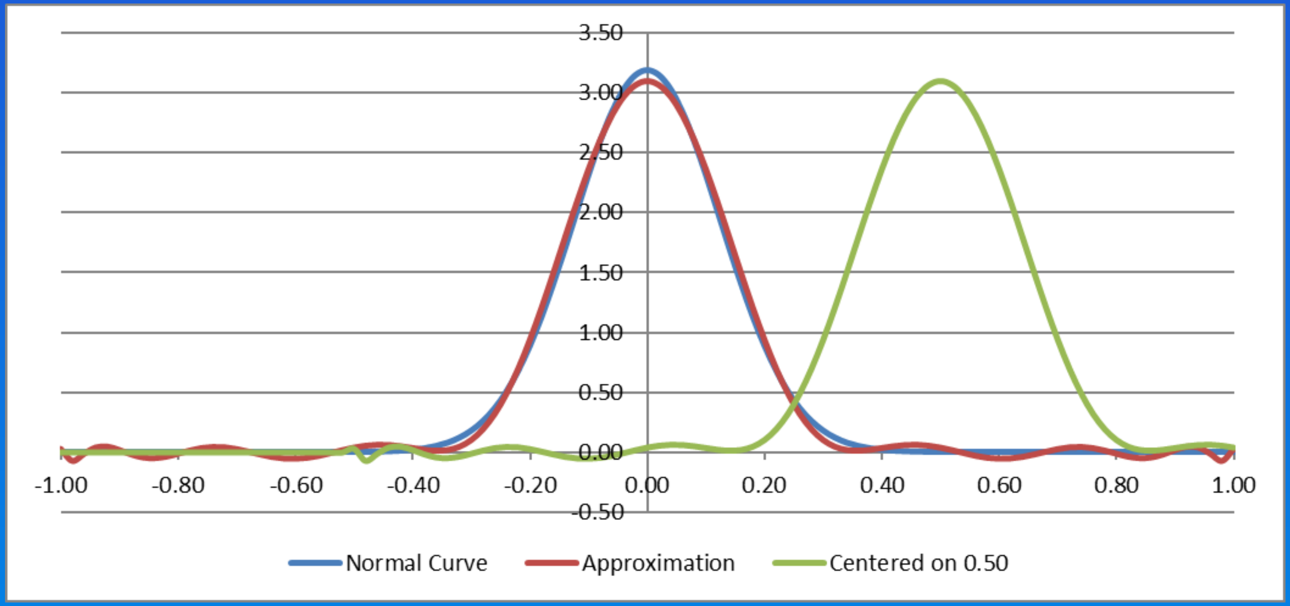
Example: A "Normal" Kernel

The function, $K(u, v)$, is a two-variable function that approximates a normal distribution with random variable u and mean v.

Following the algebra, $K(u, v) = f(u)'Kf(v)$, a quadratic form. For the example, $f(u)' = [1\ u\ u^2\ \ldots\ u^{16}]$.

The real square matrix K is derived from a statistical fit of the normal density as described in the paper.

Below: blue is the normal density with mean 0 and $\sigma = 0.125$, the red curve is the approximation of $K(u, v)$, for $v = 0$; the green curve is the approximation for $v = 0.5$. Different values of v simply slide the normal approximation horizontally.



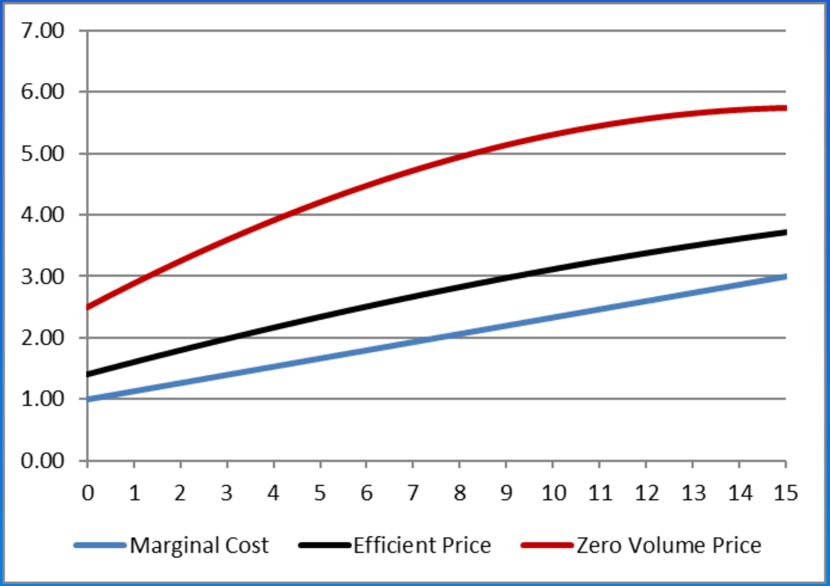
Example: Proposition 1

Proposition 1: The efficient price function is a weighted average of the marginal cost function and the zero-volume price function.

Blue: The marginal cost function. Assumed to be a linear function of weight per piece.

Red: The zero-volume price function. The linear integral equation for demand is solved for the inverse demand function which is a linear integral equation for price as a function of quantity. The inverse is then evaluated with quantity set to zero at all levels of weight per piece.

Black: The efficient price function. The weight for the average is chosen so that the efficient price function yields a pre-determined contribution to institutional cost.



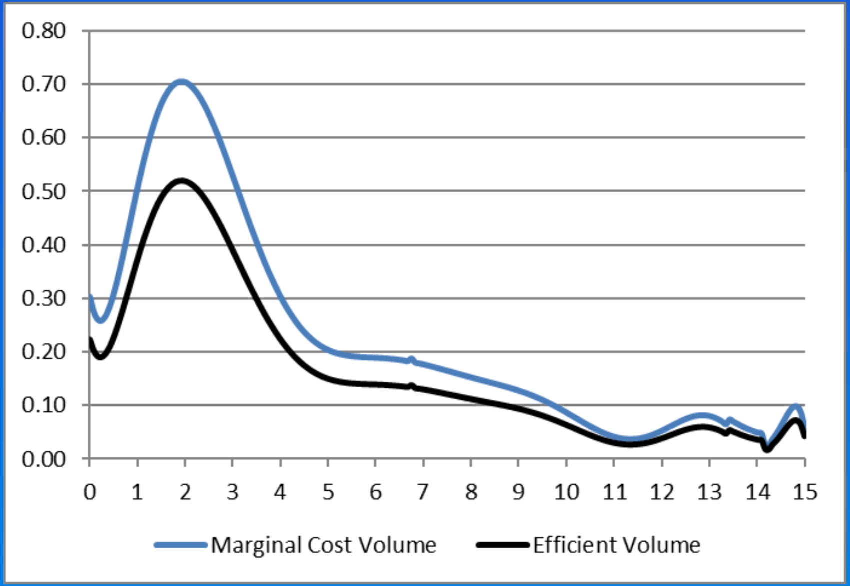
Example: Proposition 2

Proposition 2: The efficient volume function is proportional to the volume function corresponding to marginal cost pricing.

Blue: Marginal cost volume, the demand function evaluated using marginal cost as the price function.

Black: The efficient volume function. The proportion is identical to the weight used to calculate the efficient price function following Proposition 1, i.e., the proportion that yields the specified contribution to institutional cost.

The efficient price function is obtained by evaluating the inverse demand function for the efficient volume.



The Case for Efficient Rate Design

The current USPS practice is (roughly) to design rates by scaling the marginal cost function. Intuitively, this seems like a reasonable way to relate rates to an hedonic property. However, the propositions and example show that this will result in an inefficient rate design except in the rare case when $C_f = 0$.

The information requirements for applying the propositions are not any different than the information requirements of current practice. Both require estimates of the demand and cost equations.

The actual welfare gain from imposing an efficient design in any single instance is likely to be small. But a complex modern postal tariff contains many instances where rates are related to an hedonic property.

The assumption of linear forms for the integral equations for demand and cost do not seriously limit the applicability of Propositions 1 and 2. The Euler equation and the propositions derived from it must hold for linearizations of these equations in the region of the efficient $P(u)$ and $Q(u)$.

Likewise, the limits on the selection of the real vector function $f(u)$ and the range $u \in [0,1]$ are unimportant in practice. The index u and the function $f(u)$ can be chosen to approximate, with sufficient accuracy, any of the one- or two-variable functions that are likely to be encountered in any actual economic application.

In general, there appears to be little to prevent USPS and other posts from applying Propositions 1 and 2 to improve the efficiency of their tariffs.